Internal Letter

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SUBJECT: Balance and Isolation Requirements for the Phase-II Despreading Mixer Design.

I. INTRODUCTION

In the GPS receiver, the effect of the despreading mixer on the signal, thermal noise, and a CW jammer determines the A/J performance. A simple model for this type of signal processing can be used to determine the balance and isolation requirements of the double-balanced mixer design. A pictorial representation along with some practical assumptions has been chosen to avoid a long derivation of mathematical results.

II. INPUT SIGNAL STRUCTURE

If secondary effects such as doppler and phase noise on the signal and jammer modulation are ignored, the inputs to the receiver can be expressed as

$$s_o(t) = [D(t) \oplus m(t)] \, A \cos [W_0 t + \theta_o]$$

$$j(t) = (2J)^{\frac{1}{2}} \cos[W_1 t + \theta_1]$$

$$n(t) = (kT_r B_1)^{\frac{1}{2}} \{n_c(t) \cos W_o t - n_s(t) \sin W_0 t\}$$

where

$s(t)$ = Input spread spectrum signal.
$D(t)$ = 50 Baud digital data.
$m(t)$ = Direct sequence PN code.
f_0 = L-Band carrier frequency.
$j(t)$ = CW jammer signal.
J = Jammer power.

f_1 = Jammer frequency
$n(t)$ = Narrowband thermal noise.
T_r = Receiver Effective Noise Temperature.
B_1 = Bandwidth of wide bandpass filter.
n_c, n_s = Quadrature components of baseband thermal noise with zero means and unity variances.

Because the analysis applies only to the case where the receiver is being jammed, the C/A code will be ignored. This is valid for despreading analysis because the C/A code power is reduced to an insignificant level by the processing gain.

The narrow bandwidth of the carrier modulated by the digital data will be represented by a line spectra.

III. SIGNAL ANALYSIS

Figure 1 is a simplified block diagram of the despreading process. The power spectral density functions for the inputs are shown in Figure 2 where:

N_t = kT_r, the single-sided PSD function for the thermal noise. T_r is the receiver noise temperature and k is Boltzmann's constant.
Δ_t = Chip spacing for the P code.
s = $\frac{1}{2}A^2$, the signal power.
J = Jammer power.

For a properly designed spread-spectrum receiver, the RF bandwidth is chosen to pass the signal-between-first-nulls. The output spectra of such a wide bandpass filter is shown in Figure 3. The mixer multiplies the bandlimited input signals with the local code. In the frequency domain, the mixer output spectra is obtained by the convolution of the spectra of Figure 3 and the spectra of the local code. The result is shown in Figure 4. Since the mixer output will be narrowband filtered, only the spectra near f_o will be of interest. The spectra at the output of the narrow bandpass filter is shown in Figure 5. It consists of the following:

1) Thermal noise in the passband with a single-sided spectral amplitude of $N_t = kT_r$.

2) The de-spread P signal with total power of $S = \frac{1}{2}A^2$.
3) Random noise due to the spread jammer with a single-sided spectral amplitude of $J\Delta_t$.
4) A reduced-power CW jammer component due to mixer unbalance and signal feed-around. The total power of this undesired jamming component is $\alpha_o J$ where α_o is the power attenuation factor.

IV. OUTPUT SIGNAL STRUCTURE

For our simple model, the signals at the output of the narrow bandpass filter are:

$$s(t) = D(t)\, A \cos\left[W_{0t} + \theta_o\right]$$

$$j_o(t) = (2\alpha_o J)^{\frac{1}{2}} \cos\left[\omega_1 t + \theta_1\right]$$

$$n(t) = (N_0 B_2)^{\frac{1}{2}} \{n_c(t) \cos \omega_o t - n_s(t) \sin\omega_o t\}$$

where:

α_0 = Jammer power leakage attenuation factor.

N_0 = $(kT_r + J\Delta_t)$, the single-sided noise power spectral density in the narrow bandpass filter.

The despread P signal is a 50 Baud bi-phase modulated sinusoid at an RF frequency of f_0 Hz. The narrowband noise has a component due to receiver thermal noise and a component due to spreading of the CW jammer. Figure 6 shows the composite spectra of the despreading processor output.

The narrow bandpass filter bandwidth must be wide enough to pass the 50 Baud digital data signal. If the design is implemented with an integrate-and-dump matched filter, the power transfer function may be represented, at RF, by:

$$|H(jw)|^2 = \left|\frac{\sin \pi(f-f_0)\, \Delta t}{\pi(f-f_0)}\right|^2$$

The effective noise bandwidth at RF for such an implementation is:

$$B_2 = \frac{1}{\Delta t}$$

Since Δt = 20 milliseconds for a 50 Baud data rate, the noise bandwidth is 50 Hz.

The bandwidth-between-first-nulls is given by:

$$BWFN = \frac{2}{\Delta t}$$

If a filter implementation other than matched filtering is used, it is desirable that the bandwidth be no smaller than BWFN which results in a 100 Hz noise bandwidth.

V. DESIGN ANALYSIS

Using the output signal model just developed it is possible to predict the GPS receiver performance in the areas of carrier tracking, code tracking, data demodulation, and P signal reacquisition. For the purposes of this report, we will concentrate on the requirements for CW jammer carrier isolation in the despreading mixer stage.

For P signal reacquisition, the jammer leakage must not exceed the power in the desired signal. As a rule-of-thumb, the leakage level should be 5 dB below the weakest desired signal. The following conditions apply for Phase II:

$$S \geqslant -163 \text{ dBW}$$
$$\alpha_0 J \leqq S - 5 \text{ dB}$$
$$C/N_0 \geqslant +10 \text{ dB-Hz} \quad \text{(Minimum design point)}$$
$$J/S \leqq 60 \text{ dB}$$

For these conditions it is clear that the despreading mixer isolation should exceed:

$$10 \log \left[\frac{1}{\alpha_0}\right] \geqslant 65 \text{ dB} \quad \text{(Reacquisition)}$$

For carrier phase detection in the Costas loop we assume that the jammer frequency is exactly on the P signal and the I/Q outputs are:

$$I = D(t) A \cos\theta_0 + (2\alpha_0 J)^{\frac{1}{2}} \cos\theta_1$$
$$Q = D(t) A \sin\theta_1 + (2\alpha_0 J)^{\frac{1}{2}} \sin\theta_1$$

The detector output is:

$$I \cdot Q = \tfrac{1}{2}A^2 \sin 2\theta_0 + D(t) A(2\alpha_0 J)^{\frac{1}{2}} \sin(\theta_0+\theta_1) + \alpha_0 J \sin 2\theta_1$$

(see attached page)

$$I = D(t)\,A\cos\theta_0 + (2\alpha_0 J)^{\frac{1}{2}}\cos\theta_1$$
$$Q = D(t)\,A\sin\theta_0 + (2\alpha_0 J)^{\frac{1}{2}}\sin\theta_1$$

Let $\beta = \dfrac{\alpha_0 J}{\frac{1}{2}A^2}$

$$I\cdot Q = \tfrac{1}{2}A^2\left\{\sin 2\theta_0 + 2D(t)\beta\sin(\theta_0+\theta_1) + \beta^2\sin 2\theta_1\right\}$$

Tracking Condition:

$$E\{I\cdot Q\} = 0$$

Spectrum of $D(t)$ is $\dfrac{\sin x}{x}$. This causes spreading of the jammer. θ_1 may be an arbitrary function of time but θ_0 is assumed to be a fixed constant for this analysis.

The condition for carrier phase tracking is

$$I \cdot Q = 0$$

so when no jammer is present the tracking phase of the desired signal is

$$\theta_0 = 0.$$

Assuming the presence of a jammer with worst case phase angle, we can use the approximations;

$$D(t)\sin(\theta_0+\theta_1) \approx \sin 2\theta_1 = 1$$

this is only valid when the jammer flips phase with the signal. Otherwise there is a spreading effect

and the substitution;

$$\beta^2 = \frac{\alpha_0 J}{\frac{1}{2}A^2} = \alpha_0 (J/S)$$

to obtain the approximate solution:

$$I \cdot Q \approx \tfrac{1}{2}A^2 \{ \sin 2\theta_0 + 2\beta + \beta^2 \}$$

For the small phase errors required for good tracking performance we can assume small β and θ_0 and use small angle approximations to obtain the tracking conditions:

$$I \cdot Q \approx A^2 \{ \hat{\theta}_0 + \beta \}$$

$$\hat{\theta}_0 \approx -\beta$$

For the jammer leakage to cause a peak error no larger than 5 degrees, the following conditions must be satisfied:

$$\beta \leq 2\pi \left(\frac{5}{360}\right)$$

In terms of the jammer leakage attenuation factor, this requirement becomes:

$$\frac{1}{\alpha_0} \geqslant \left[2\pi\left(\frac{5}{360}\right)\right]^{-2} (J/S)$$

For carrier tracking to J/S = 53 dB, the despreading mixer isolation should exceed:

$$10 \log \left[\frac{1}{\alpha_0} \right] \geqslant 74.2 \text{ dB} \quad \text{(Carrier Tracking)}$$

This level of performance would also satisfy the requirements for data demodulation.

For delay-lock code tracking, the jammer leakage contributes to the RMS tracking error. To eliminate this effect, the leakage power should be masked by the noise in the predetection bandwidth. This requirement is expressed as:

$$\alpha_0 J << N_0 B_2$$

or

$$\alpha_0 << \Delta_t B_2$$

Assuming a level of 10 dB into the noise and a predetection noise bandwidth of 50 Hz., the requirement becomes:

$$10 \log \left[\frac{1}{\alpha_0}\right] \geqslant 63.1 \text{ dB} \quad \text{(Delay-lock code tracking)}$$

A similar analysis for the (E-L) tracking loop has not been performed.

VI. SUMMARY

The requirements on the jammer leakage attenuation factor for good despreading mixer design have been analyzed. The most demanding requirement was found to be carrier phase tracking. The leakage attenuation factor must meet the requirement;

$$\frac{1}{\alpha_0} \geqslant \left[2\pi \frac{\hat{\theta}}{360}\right]^{-2} (J/S)_{MAX}$$

where:

$\hat{\theta}$ = Largest allowable peak phase tracking error in degrees.

$(J/S)_{MAX}$ = Tracking limit of the phase tracking loop.

For 10 log (J/S) = 53 dB and a peak phase error less than 5 degrees, the leakage attenuation factor should exceed 74.2 dB.

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att

ljc

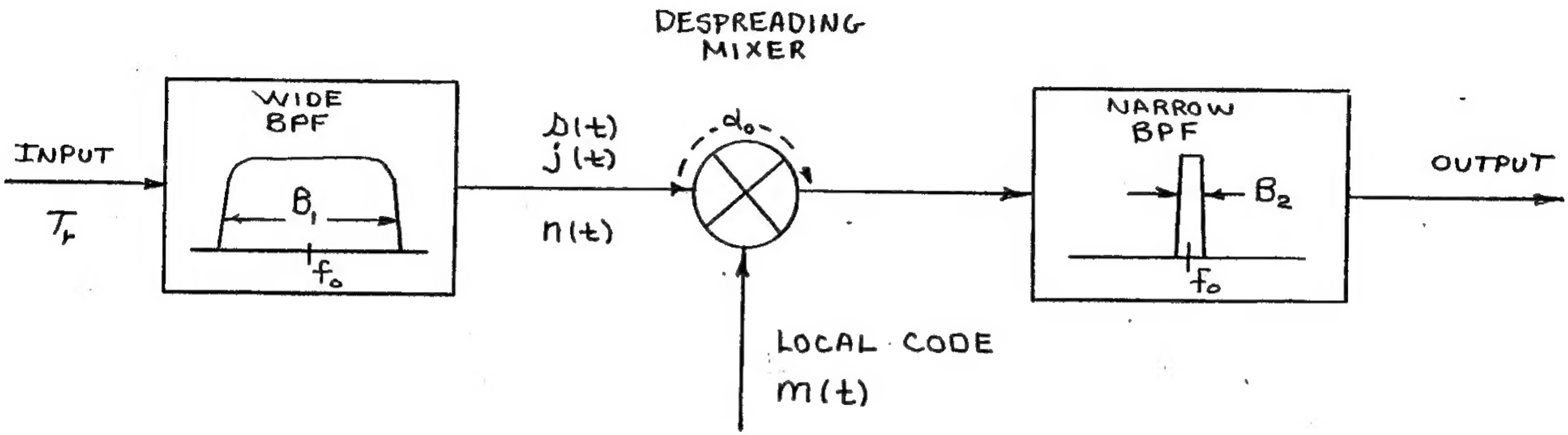

FIGURE 1. A Simplified Block Diagram of the Despreading Process.

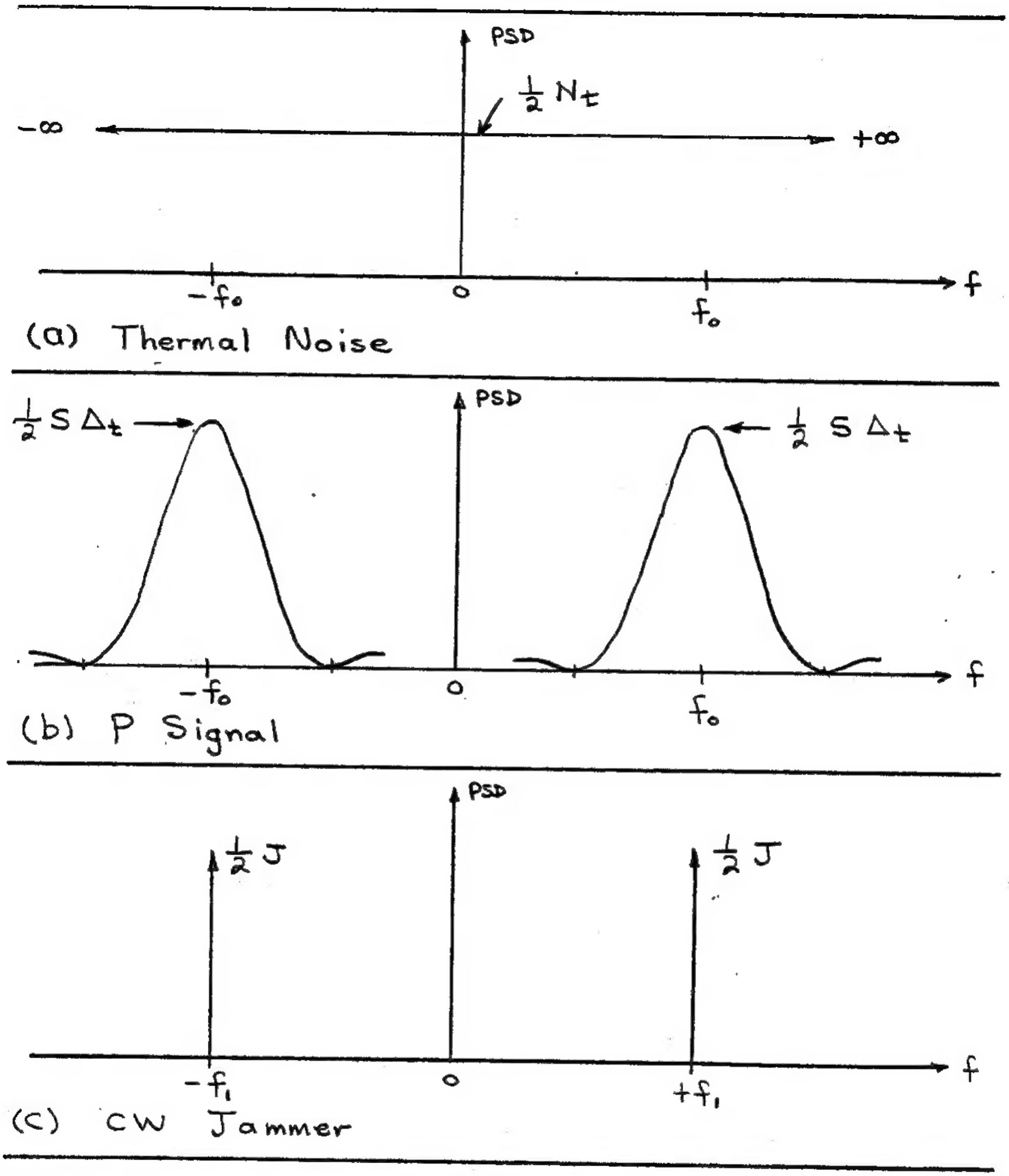

FIGURE 2. Input Power Spectra.

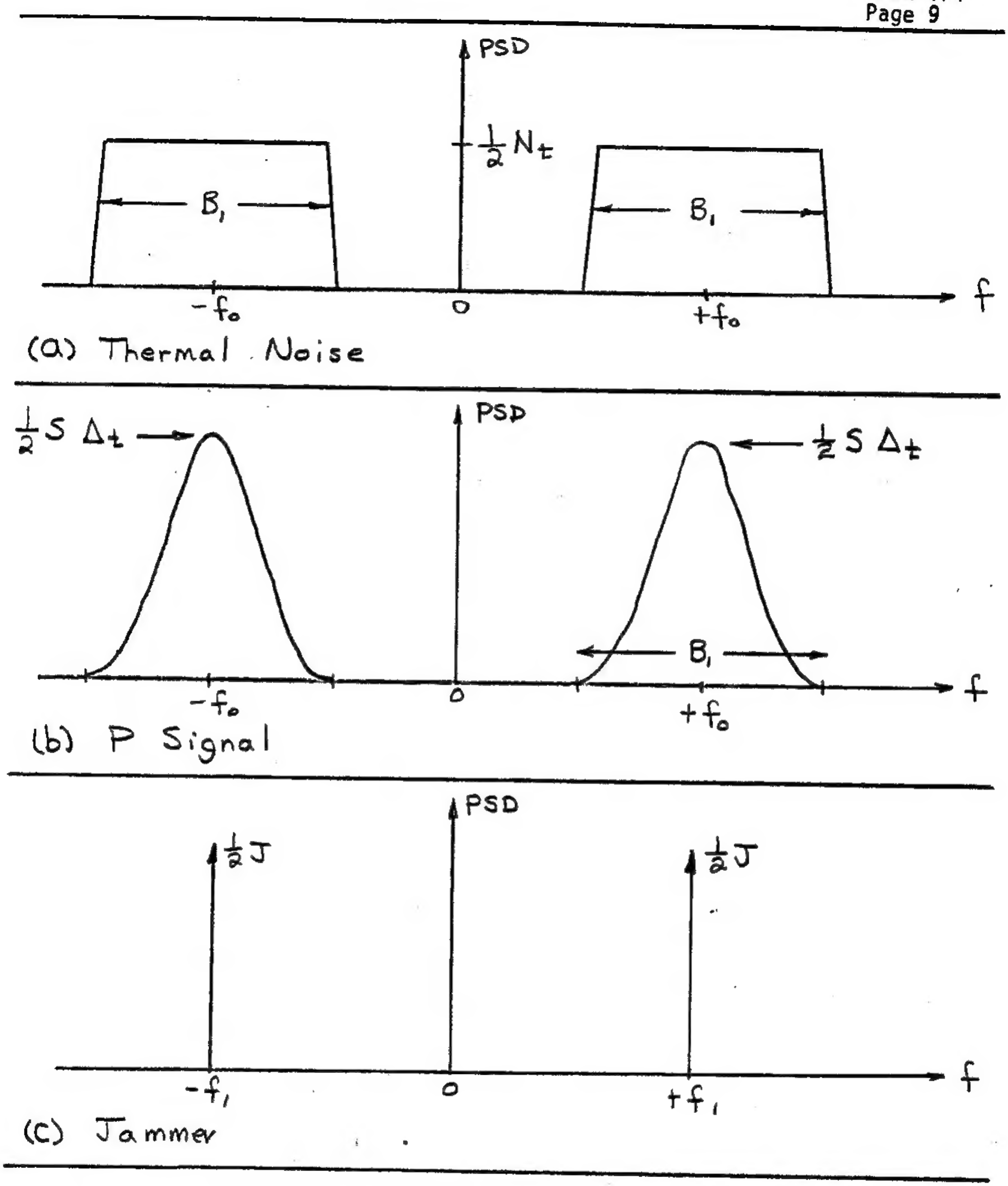

FIGURE 3. Power Spectra at the Mixer Input.

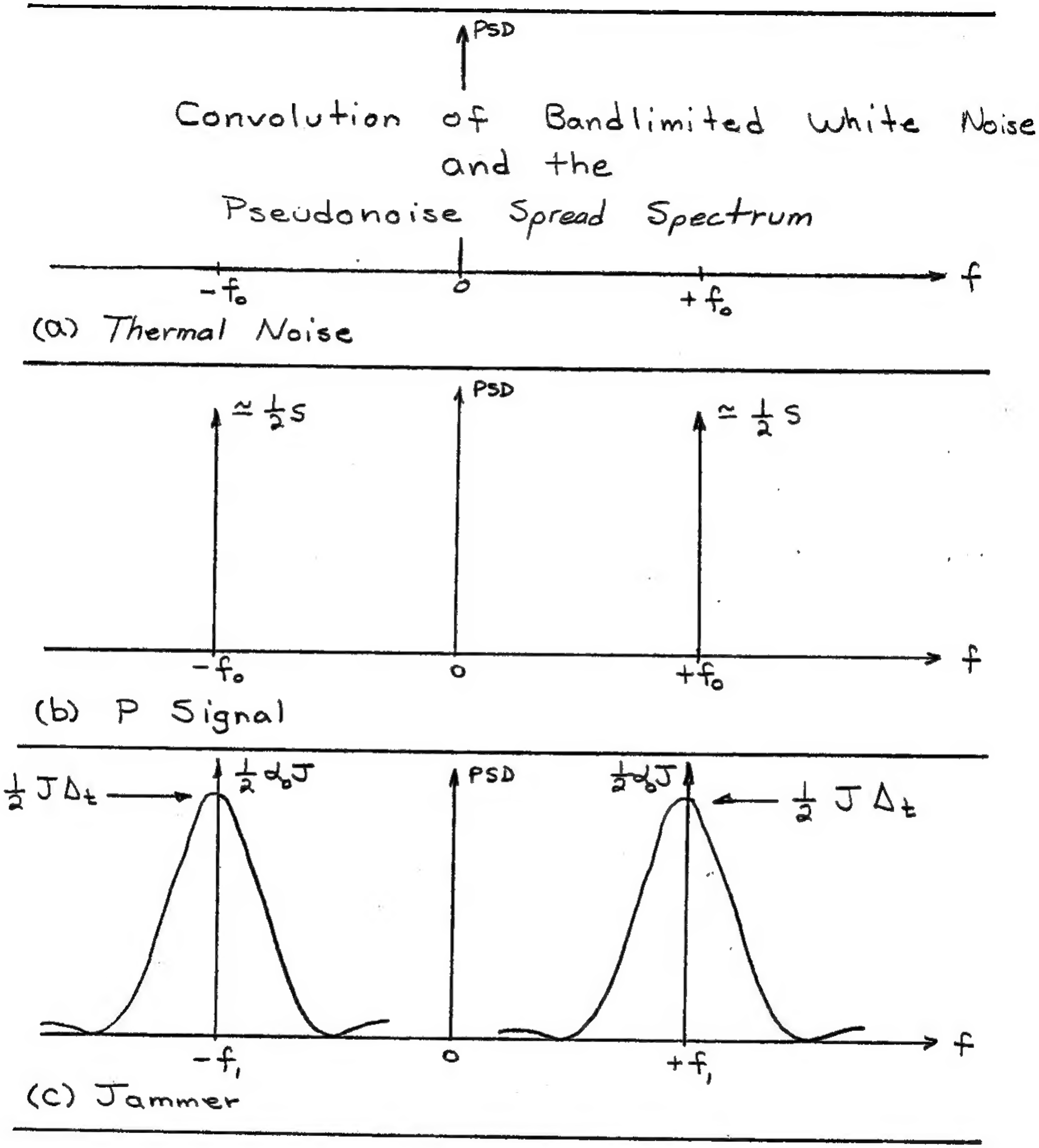

FIGURE 4. Power Spectra at the Mixer Output.

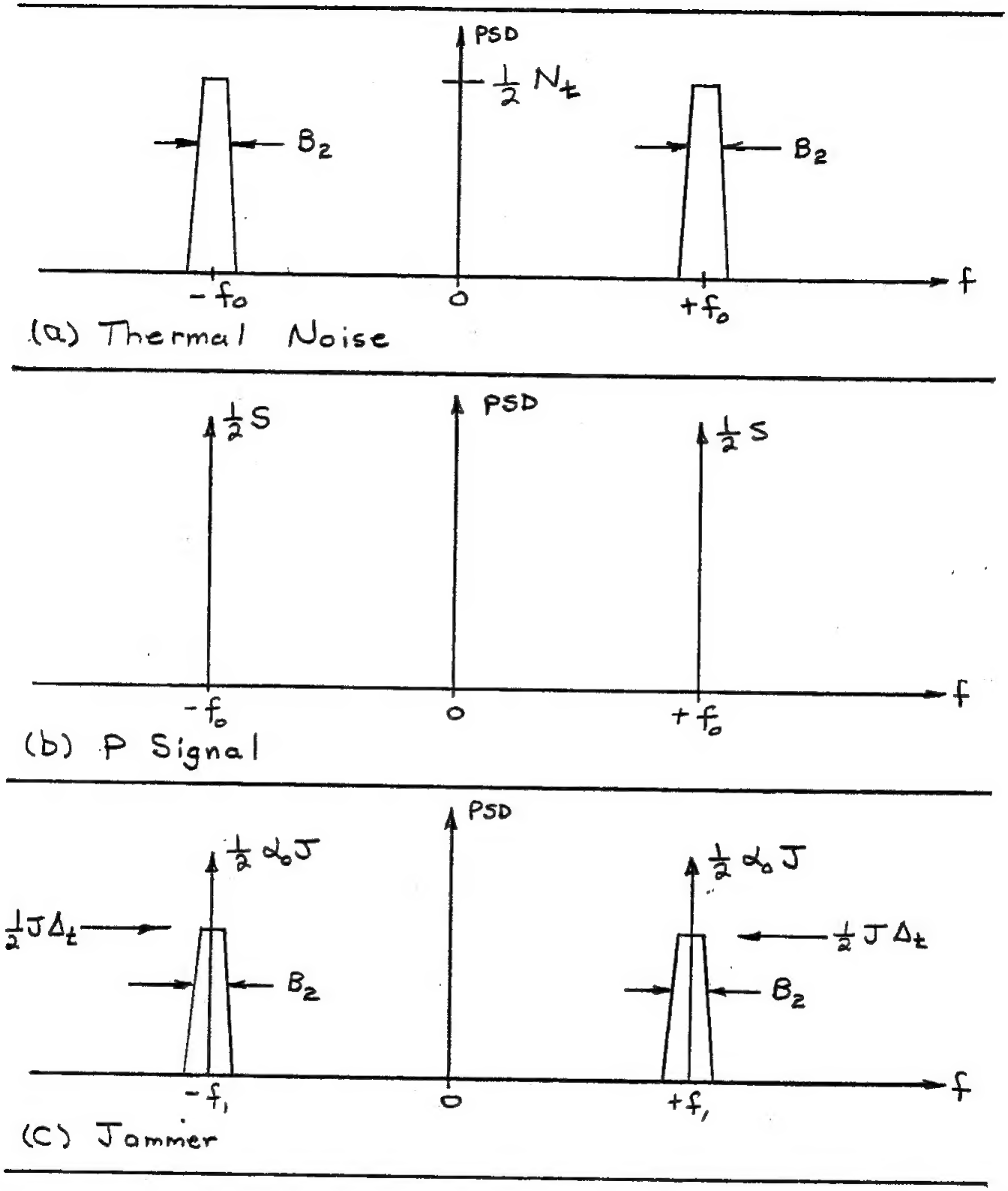

FIGURE 5. Power Spectra at the Narrow Bandpass Filter Output.

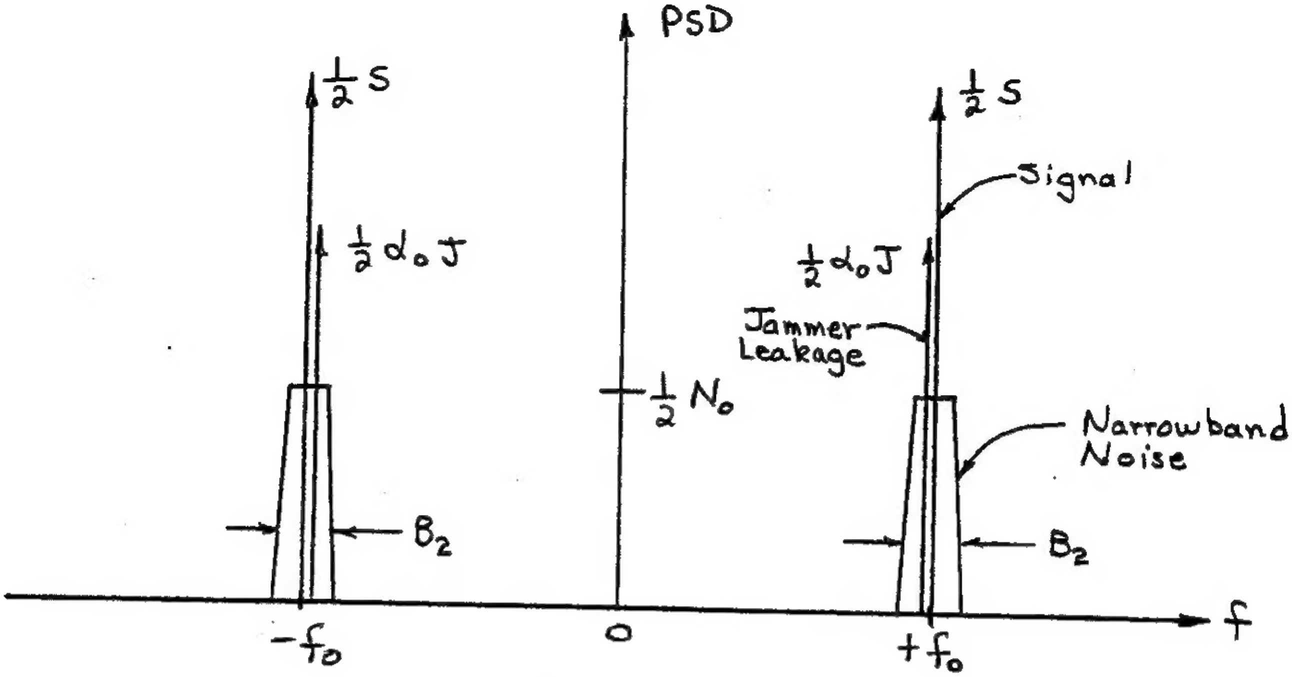

FIGURE 6. Composite Output Spectra.